

Chemical Engineering Thermodynamics
Quiz 3 January 30, 2020

Consider an idealized engine cycle modeled after the Carnot cycle involving **5 moles** of nitrogen with a heat capacity, $C_p = 3.5R$ (ideal gas). The initial condition (same as stage 4) is 500 K and 5 MPa which is labeled Stage 0 in the table below. Use stage 4 as the reference state where $S = 0$. (1 point each total 50)

-Make sure you include the number of moles in your calculations.

-Make sure to give the results in kJ not in J.

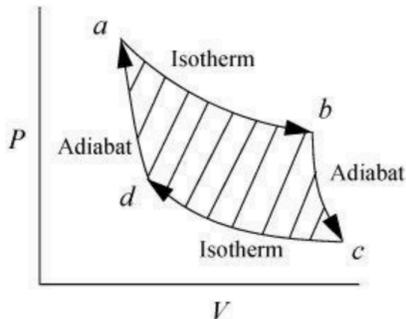
-For stage 3, calculate ΔS_3 and ΔS_4 first.

		Stage	Moles	T , K	P , Mpa	V , cm ³	W_{ec} , kJ	Q , kJ	ΔU , kJ	ΔS , kJ/K	ΔH , kJ	S , kJ/K
		0	5	500	5							
Expansion	Isothermal	1			4							
	Adiabatic	2			0.1							
Compression	Isothermal	3										
	Adiabatic	4		500	5							0
		NET								0		

b) Sketch T versus S for this cycle using four the values you calculated and show how you can obtain Q_{Hot} and Q_{Cold} from this plot. (12.5 points)

c) On the plot of T versus S show how a cycle of 1) isothermal, 2) isochoric, 3) isothermal, 4) isochoric would appear (part a of homework problem 3.2). In general terms how would the Net values in the table change in going from adiabatic to isochoric stages 2 and 4? (12.5 points)

d) In the text the following plot is shown for the Carnot Cycle.



Comment on how this plot compares with the values you obtained, that is, if you make a plot of your PV values does it appear similar to this plot? (12.5 points)

e) Calculate the efficiency of your process from the net work compared to the heat input. How does this compare with the theoretical efficiency for a Carnot Cycle using the temperatures? (12.5 points)

1 atmosphere is 14.7 psi, 1.01 bar, 0.101 MPa, 760 mmHg, 29.9 inHg

Gas Constant, R

$$\begin{aligned}
 &= 8.31447 \text{ J/mole-K} = 8.31447 \text{ cm}^3\text{-MPa/mole-K} = 8.31447 \text{ m}^3\text{-Pa/mole-K} \\
 &= 8,314.47 \text{ cm}^3\text{-kPa/mole-K} = 83.1447 \text{ cm}^3\text{-bar/mole-K} = 1.9859 \text{ Btu/lbmole-R}^{(\text{see note 1})} \\
 &= 82.057 \text{ cm}^3\text{-atm/mole-K} = 1.9872 \text{ cal/mole-K}^{(\text{see note 2})} = 10.731 \text{ ft}^3\text{-psia/lbmole-R}
 \end{aligned}$$

Process Type	Work Formula (ig)
Isothermal	$W_{EC} = -\int P dV = -RT \int \frac{dV}{V} = -RT \ln \frac{V_2}{V_1}$ (ig)
Isobaric	$W_{EC} = -\int P dV = -P(V_2 - V_1)$ (ig)
Adiabatic and reversible	$W_{EC} = -\int P dV = -\int \text{const} \frac{dV}{V^{(C_p/C_v)}}$ (*ig) or $\Delta U = C_v(T_2 - T_1) = W_{EC}$ (*ig) $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(R/C_p)} = \left(\frac{V_1}{V_2}\right)^{(R/C_v)}$ (*ig)

$$Q_{\text{rev}} = \Delta U \text{ for isochoric (constant volume)}$$

4.17

$$dU = C_v dT \text{ for isochoric (constant volume)}$$

$$C_p = C_v + R \text{ (exact for ideal gas)}$$

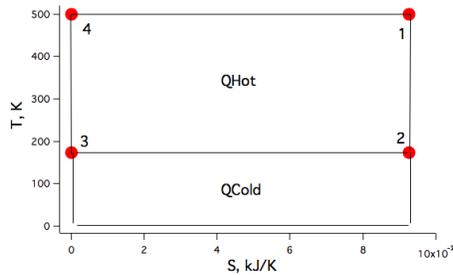
$$\Delta H = \Delta U + \Delta(PV) = \Delta U + R(\Delta T) \text{ (exact for ideal gas)}$$

$$T_2/T_1 = (P_2/P_1)^{R/C_p} = (V_1/V_2)^{R/C_v}$$

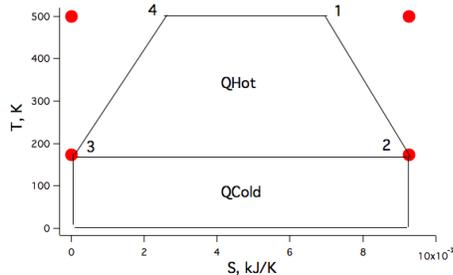
**ANSWERS: Chemical Engineering Thermodynamics
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		Stage	Moles	T , K	P , Mpa	V , cm ³	W_{ec} , kJ	Q , kJ	ΔU , kJ	ΔS , kJ/K	ΔH , kJ	S , kJ/K
		0	5	500	5	4160						
Expansion	Isothermal	1	5	500	4	5190	-4.64	4.64	0	0.00927	0	0.00927
	Adiabatic	2	5	174	0.1	72400	-33.9	0	-33.9	0	-47.4	0.00927
Compression	Isothermal	3	5	174	0.125	57800	1.61	-1.61	0	-0.00927	0	0
	Adiabatic	4	5	500	5	4160	33.9	0	33.9	0	47.4	0
		NET					-3.03	3.03	0	0	0	

b)

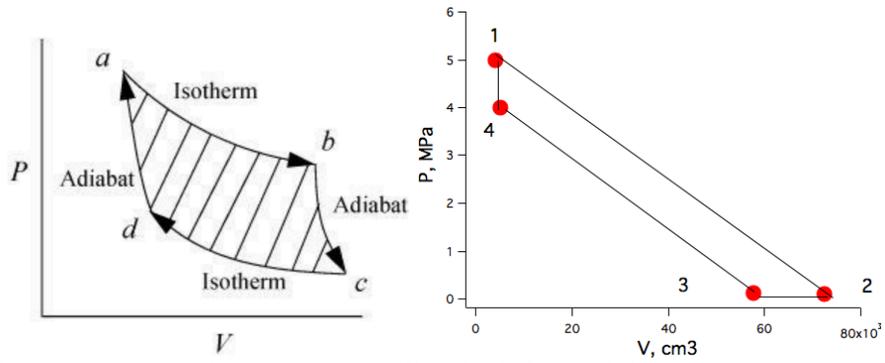


c)



ΔS would have a net positive value, ΔH and ΔU would have net value 0. W_{ec} would not be equal to negative Q .

d)



The two graphs are not very close. It seems like the left graph isn't a very good representation of the Carnot process.

$$e) -W_{ec\ net}/Q_{I\ hot} = 3.03\text{ kJ}/4.64\text{ kJ} = 0.653$$

$$(T_H - T_C)/T_H = (500\text{K} - 174\text{K})/500\text{K} = 0.652$$

The two are the same within rounding errors.

Stage 0

$$V_4 = V_0 = \frac{nRT}{P} = \frac{5 \text{ mol} \times 0.31 \frac{\text{m}^3 \text{ mol}^{-1} \text{ s}^{-1} \text{ K}}{\text{mol} \cdot \text{K}} \times 500 \text{ K}}{5 \text{ MPa}} = 4,160 \text{ cm}^3$$

Stage 1

Isobaric

$$V_1 = \frac{nRT}{P} = \frac{5 \text{ mol} \times 0.31 \frac{\text{m}^3 \text{ mol}^{-1} \text{ s}^{-1} \text{ K}}{\text{mol} \cdot \text{K}} \times 500 \text{ K}}{4 \text{ MPa}} = V_0 \frac{5 \text{ MPa}}{4 \text{ MPa}} = 5,190 \text{ cm}^3$$

$$\Delta H = \Delta U = 0$$

$$W_{EC} = -\int P dV = -R T \ln \frac{V_2}{V_1} = +R T \ln \frac{P_2}{P_1} = \left(0.31 \frac{\text{m}^3 \text{ mol}^{-1} \text{ s}^{-1} \text{ K}}{\text{mol} \cdot \text{K}} \times 500 \text{ K} \ln \frac{4}{5}\right) 5 \text{ mol}$$

$$= -927 \frac{\text{J}}{\text{mol}} \cdot 5 \text{ mol} = -4.64 \text{ kJ}$$

$$Q = -W_{EC} = 4.64 \text{ kJ}$$

$$\Delta S = \frac{Q}{T} = \frac{4.64 \text{ kJ}}{500 \text{ K}} = 0.00927 \frac{\text{kJ}}{\text{K}}$$

Stage 2

Adiabatic

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\gamma/\gamma-1} = 500 \text{ K} \left(\frac{0.1}{4}\right)^{1/3.5} = 174 \text{ K}$$

$$V_2 = \frac{nRT}{P} = \frac{5 \text{ mol} \times \left(0.31 \frac{\text{m}^3 \text{ mol}^{-1} \text{ s}^{-1} \text{ K}}{\text{mol} \cdot \text{K}}\right) (174 \text{ K})}{0.1 \text{ MPa}} = 72,400 \text{ cm}^3$$

$$Q = 0$$

$$\Delta U = W_{EC} = n C_v (T_2 - T_1) = 5 \text{ mol} \times \left(2.5 \times 0.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) (174 \text{ K} - 500 \text{ K})$$

$$= -33,900 \text{ J} = -33.9 \text{ kJ}$$

$$\Delta H = n C_p (T_2 - T_1) = \Delta U \left(\frac{3.5}{2.5}\right) = -47.9 \text{ kJ}$$

Stage 3

Isobaric

$$\Delta H = \Delta U = 0$$

$$\Delta S_{NET} = 0 \quad \Delta S_2 = \Delta S_4 = 0 \quad \Delta S_3 = -\Delta S_1 = -0.00927 \frac{\text{kJ}}{\text{K}}$$

$$Q_3 = \Delta S_3 T = -0.00927 \frac{\text{kJ}}{\text{K}} (174 \text{ K}) = -1.61 \text{ kJ} = -W_{EC}$$

$$W_{EC} = nRT \ln \frac{P_2}{P_1}$$

$$P_3 = P_2 \exp\left(\frac{W_{EC}}{nRT_3}\right) = 0.1 \text{ MPa} \exp\left(\frac{1.61 \text{ kJ} \times 1000 \text{ J/kJ}}{5 \text{ moles} \times 8.31 \frac{\text{J}}{\text{mol K}} \times 174 \text{ K}}\right) = 0.125 \text{ MPa}$$

$$V_3 = \frac{nRT_3}{P_3} = \frac{5 \text{ moles} \times 8.31 \frac{\text{J}}{\text{mol K}} \times 174 \text{ K}}{0.125 \text{ MPa}}$$

$$V_3 = 57,800 \text{ cm}^3$$

Step 4
adiabatic

$$Q = 0 \quad \Delta S = 0$$

$$\begin{aligned} \Delta U &= W_{EC} = n C_v (T_4 - T_3) \\ &= 5 \text{ moles} \left(2.5 \left(8.31 \frac{\text{J}}{\text{mol K}}\right)\right) (500 \text{ K} - 174 \text{ K}) \\ &= 33,900 \text{ J} = 33.9 \text{ kJ} \end{aligned}$$

$$\Delta H = \Delta U \frac{C_p}{C_v} = 33.9 \text{ kJ} \frac{3.5}{2.5} = 47.4 \text{ kJ}$$

NET $\Delta H_{net} = \Delta U_{net} = 0$ State function
No Loss

$$Q_{net} = 3.03 \text{ kJ} = -W_{EC, net} \quad \text{No Loss} \quad W_{net} \leftrightarrow H_{net}$$

e) $\eta = \frac{-W_{EC, net}}{Q_1 (Hot)} = \frac{3.03 \text{ kJ}}{4.64 \text{ kJ}} = 0.653$

$$\eta = \frac{T_H - T_C}{T_H} = \frac{500 \text{ K} - 174 \text{ K}}{500 \text{ K}} = 0.652$$